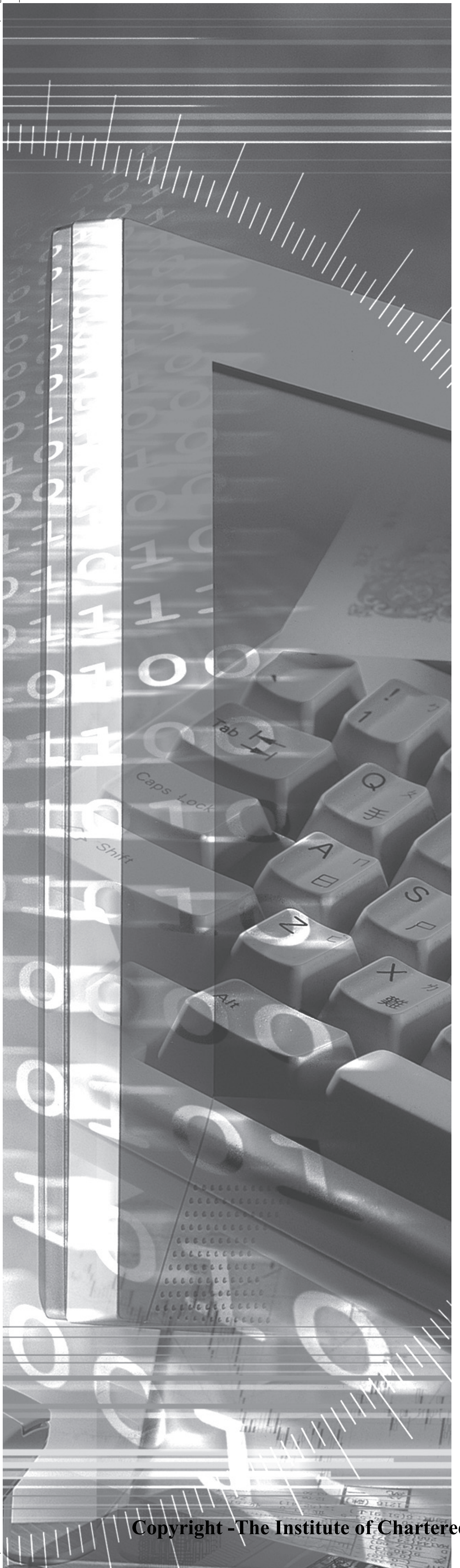

CHAPTER – 8

LIMITS AND CONTINUITY-INTUITIVE APPROACH

LEARNING OBJECTIVES

After studying this chapter, you will be able to:

- Know the concept of limits and continuity;
- Understand the theoruems underlying limits and their applications; and
- Know how to solve the problems relating to limits and continuity with the help of given illustrations.

8.1 INTRODUCTION

Intuitively we call a quantity y a function of another quantity x if there is a rule (method procedure) by which a unique value of y is associated with a corresponding value of x.

A function is defined to be rule that associates to any given number x a single number f(x) to be read as function of x. f(x) does not mean f times x. It means given x, the rule f results the number f(x).

Symbolically it may be written in the form y = f(x).

In any mathematical function y = f(x) we can assign values for x arbitrarily; consequently x is the independent variable while the variable y is dependent upon the values of the independent variable and hence dependent variable.

Example 1: Given the function f(x) = 2x + 3 show that f(2x) = 2 f(x) – 3.

Solution: LHS. f(2x) = 2 (2x) + 3 = 4x + 6 – 3 = 2(2x + 3) – 3

= 2 f(x) – 3.

Example 2: If $f(x) = ax^2 + b$ find $\frac{f(x+h)-f(x)}{h}$.

Solution:

$$\frac{f(x+h)-f(x)}{h} = \frac{a(x+h)^2+b-ax^2-b}{h} = \frac{a\,(x^2+2xh+h^2-x^2)}{h} = \frac{h\,a(2x+h)}{h}$$

= a(2x + h)

Note: f (x) = | x – a | means f (x) = x – a for x > a

= a – x for x < a.

= x – a for x = a

Example 3: If f(x) = |x| + |x – 2| then redefine the function. Hence find f (3.5), f (– 2), f(1.5).

Solution:

If x > 2 f (x) = x + x – 2 = 2x – 2

If x < 0 f (x) = – x – x + 2 = 2 – 2x

If $0 \le x \le 2$. f (x) = x – x + 2 = 2

So the given function can be redefined as f(x) = 2 – 2x for x < 0

= 2 for 0 ≤ x ≤ 2

= 2x – 2 for x > 2

for x = 3.5 f (x) = 2(3.5) – 2 = 5 , f (3.5) = 5

for x = – 2 f (x) = 2 – 2(– 2) = 6 f (–2) = 6

for x = 1.5 f (x) = 2. f (1.5) = 2

Note. Any function becomes undefined (i.e. mathematically cannot be evaluated) if denominator is zero.

Example 4: If $f(x) = \dfrac{x+1}{x^2-3x-4}$ find f(0), f(1), f(– 1).

Solution: $f(x) = \dfrac{x+1}{(x-4)(x+1)}$ $\therefore f(0) = \dfrac{1}{-4} = \dfrac{-1}{4}$, $f(1) = \dfrac{2}{(-3)(2)} = \dfrac{1}{3}$ $f(-1) = \dfrac{0}{0}$ which is not possible

i.e. it is undefined.

Example 5: If f(x) = x^2 – 5 evaluate f(3), f(–4), f(5) and f(1)

Solution: f(x) = x^2 – 5

f(3) = 3^2 – 5 = 9 – 5 = 4

f(–4) = $(–4)^2$ – 5 = 16 – 5 = 11

f(5) = 5^2 – 5 = 25 – 5 = 20

f(1) = 1^2 – 5 = 1 – 5 = – 4

8.2 TYPES OF FUNCTIONS

Even and odd functions : if a function f(x) is such that f(–x) = f(x) then it is said to be an even function of x.

Examples : f (x) = $x^2 + 2x^4$

f (–x) = $(–x)^2 + 2(–x)^4 = x^2 + 2x^4$ = f(x)

Hence f(x) = $x^2 + 2x^4$ is an even function.

On the other hand if f(x) = – f(x) then f(x) is said to be an odd function.

Examples : f (x) = $5x + 6x^3$

f (-x) = $5(-x) + 6(-x)^3 = -5x - 6x^3 = -(5x + 6x^3)$

Hence $5x + 6x^3$ is an odd function.

Periodic functions: A function f (*x*) in which the range of the independent variable can be separated into equal sub intervals such that the graph of the function is the same in each part then it is periodic function. Symbolically if f(x + p) = f(x) for all x, then p is the period of f.

Inverse function: If y = f(x) defined in an interval (a, b) is a function such that we express x as a function of y say x = g(y) then g(y) is called the inverse of f(x)

Example: i) if $y=\frac{5x+3}{2x+9}$, then $x=\frac{3-9y}{2y-5}$ is the inverse of the first function.

ii) $x=\sqrt[3]{y}$ is the inverse function of $y=x^3$.

Composite Function: If y = f(x) and x = g(u) then y = f {g(u)} is called the function of a function or a composite function.

Example : If a function f(x) = log $\frac{1+x}{1-x}$ prove that $f(x_1) + f(x_2) = f\left(\frac{x_1+x_2}{1+x_1x_2}\right)$

Solution : $f(x_1)+f(x_2) = \log\frac{1+x_1}{1-x_1} + \log\frac{1+x_2}{1-x_2}$

$$= \log\frac{1+x_1}{1-x_1}\times\frac{1+x_2}{1-x_2}$$

$$= \log\frac{1+x_1+x_2+x_1x_2}{1-x_1-x_2+x_1x_2} = \log\frac{1+\frac{x_1+x_2}{1+x_1x_2}}{1-\frac{x_1+x_2}{1+x_1x_2}} = f\left(\frac{x_1+x_2}{1+x_1x_2}\right).\ \text{Proved}$$

Exercise 8(A)

Choose the most appropriate option (a) (b) (c) or (d)

1. Given the function $f(x) = x^2 - 5$, $f(\sqrt{5})$ is equal to

   a) 0 b) 5 c) 10 d) none of these

2. If $f(x)=\frac{5^x+1}{5^x-1}$ then f(x) is

   a) an even function
   b) an odd function
   c) a composite function
   d) none of these

3. If $g(x) = 3 - x^2$ then g(x) is

a) an odd function
b) a periodic function
c) an even function
d) none of these

4. If $f(x)=\frac{q\times(x-p)}{(q-p)}+\frac{p\times(x-q)}{(p-q)}$ then f(p) + f(q) is equal to

a) p +q
b) f(pq)
c) f(p – q)
d) none of these

5. If $f(x) = 2x^2 - 5x + 4$ then 2f(x) = f(2x) for

a) x=1
b) x = – 1
c) x = ± 1
d) none of these

6. If f(x) = logx (x > 0) then f(p) + f(q) +f(r) is

a) f(pqr)
b) f(p)f(q)f(r)
c) f(1/pqr)
d) none of these

7. If $f(x) = 2x^2 - 5x + 2$ then the value of $\frac{f(4+h)-f(4)}{h}$ is

a) 11 – 2h
b) 11 + 2h
c) 2h – 11
d) none of these

8. If $y=h(x)=\frac{px-q}{qx-p}$ then x is equal to

a) h(1/y)
b) h (–y)
c) h(y)
d) none of these

9. If $f(x) = x^2 - x$ then f(h+1) is equal to

a) f(h)
b) f(–h)
c) f(–h + 1)
d) none of these

10. If $f(x)=\frac{1-x}{1+x}$ then f (f(1/x)) is equal to

a) 1/x
b) x
c) –1/x
d) none of these

8.3 CONCEPT OF LIMIT

I) We consider a function f(x) = 2x. If x is a number approaching to the number 2 then f(x) is a number approaching to the value 2 × 2 = 4.

The following table shows f(x) for different values of x approaching 2

x	f(x)
1.90	3.8
1.99	3.98
1.999	3.998
1.9999	3.9998
2	4

Here x approaches 2 from values of x<2 and for x being very close to 2 f(x) is very close to 4. This situation is defined as left-hand limit of f(x) as x approaches 2 and is written as lim f(x) = 4 as $x \to 2-$

Next

x	f(x)
2.0001	4.0002
2.001	4.002
2.01	4.02
2.0	4

Here x approaches 2 from values of x greater than 2 and for x being very close to 2 f(x) is very close to 4. This situation is defined as right–hand limit of f(x) as x approaches 2 and is written as lim f(x) = 4 as $x \to 2+$

So we write

$\lim_{x\to 2-} f(x) = \lim_{x\to 2+} f(x) = 4$

Thus $\lim_{x\to a} f(x)$ is said to exist when both left-hand and right-hand limits exists and they are equal. We write as

$$\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x) = \lim_{x\to a} f(x)$$

Thus, if $\lim_{h\to o} f(a+h) = \lim_{h\to o} f(a-h)$, (h>o)

then $\lim_{x\to a} f(x)$ exists

We now consider a function defined by

$$f(x) = \begin{cases} 2x-2 & \text{for } x<0 \\ 1 & \text{for } x=0 \\ 2x+2 & \text{for } x>0 \end{cases}$$

We calculate limit of f(x) as x tend to zero. At x = 0 f(x) = 1 (given). If x tends to zero from left-hand side for the value of x<0 f(x) is approaching (2×0) –2 = –2 which is defined as left-hand limit of f(x) as $x \to 0-$ we can write it as

Thus $\lim_{x\to 0-} = -2$

Similarly if x approaches zero from right-hand side for values of x>0 f(x) is approaching 2 × 0 + 2 = 2. We can write this as $\lim_{x\to 0+} f(x) = 2$.

In this case both left-hand limit and right-hand exist but they are not equal. So we may conclude that $\lim_{x \to 0} f(x)$ does not exist.

8.4 USEFUL RULES OF THEOREMS ON LIMITS

Let $\lim_{x \to a} f(x) = \ell$ and $\lim_{x \to a} g(x) = m$

where ℓ and m are finite quantities

i) $\lim_{x \to a} \{f(x) + g(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = \ell + m$

That is limit of the sum of two functions is equal to the sum of their limits.

ii) $\lim_{x \to a} \{f(x) - g(x)\} = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = \ell - m$

That is limit of the difference of two functions is equal to difference of their limits.

iii) $\lim_{x \to a} \{f(x) \,.\, g(x)\} = \lim_{x \to a} f(x) \,.\, \lim_{x \to a} g(x) = \ell m$

That is limit of the product of two functions is equal to the product of their limits.

iv) $\lim_{x \to a} \{f(x)/g(x)\} = \{\lim_{x \to a} f(x)\}/\{\lim_{x \to a} g(x)\} = \ell/m$

That is limit of the quotient of two functions is equal to the quotient of their limits.

v) $\lim_{x \to a} c = c$ where c is a constant

That is limit of a constant is the constant.

vi) $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

vii) $\lim_{x \to a} F\{f(x)\} = F\{\lim_{x \to a} f(x)\} = F(l)$

viii) $\lim_{x \to 0+} \frac{1}{x} = \lim_{h \to 0} \frac{1}{h} \to +\infty$ (h>0)

$\lim_{x \to 0^-} \frac{1}{x} = \lim_{h \to 0} \frac{1}{-h} \to -\infty$ (h>0)

∞ is a very-very large number called infinity

Thus $\lim_{x \to 0} 1–x$ does not exist.

Example 1: Evaluate: (i) $\lim_{x\to 2}(3x+9)$; (ii) $\lim_{x\to 5}\frac{1}{x-1}$ (iii) $\lim_{x\to a}\frac{1}{x-a}$

Solution: (i) $\lim_{x\to 2}(3x+9) = 3.2+9 = (6+9) = 15$

(ii) $\lim_{x\to 5}\frac{1}{x-1} = \frac{1}{5-1} = \frac{1}{4}$

(iii) $\lim_{x\to a}\frac{1}{x-a}$ does not exist, $\lim_{x\to a+}\frac{1}{x-a} \to +\infty$ and $\lim_{x\to a^-}\frac{1}{x-a} \to -\infty$

[Hint: L.H.S. = $\lim_{h\to 0}$ and $\lim_{h\to 0}\left(\frac{1}{-h}\right)$ (h>o)

Example 2: Evaluate $\lim_{x\to 2}\frac{x^2-5x+6}{x-2}$.

Solution: At x = 2 the function becomes undefined as 2-2 = 0 and division by zero is not mathematically defined.

So $\lim_{x\to 2}\{x^2-5x+6/(x-2)\} = \lim_{x\to 2}\{(x-2)(x-3)/(x-2)\} = \lim_{x\to 2}(x-3)$ ($\because$ x-2 ≠ 0)

= 2-3 = -1

Example 3: Evaluate $\lim_{x\to 2}\frac{x^2+2x-1}{\sqrt{x^2+2}}$.

Solution:

$$\lim_{x\to 2}\frac{x^2+2x-1}{\sqrt{x^2+2}} = \frac{\lim_{x\to 2}(x^2+2x-1)}{\lim_{x\to 2}\sqrt{x^2+2}} = \frac{\lim_{x\to 2}x^2+\lim_{x\to 2}2x-1}{\sqrt{\lim_{x\to 2}x^2+2}}$$

$$= \frac{(2)^2+2\times 2-1}{\sqrt{(2)^2+2}} = \frac{7}{\sqrt{6}}$$

8.5 SOME IMPORTANT LIMITS

We now state some important limits

a) $\lim_{x\to 0}\frac{(e^x-1)}{x}=1$

b) $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$ (a>0)

c) $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$

d) $\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^x=e$ or $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}}{x}=e$

e) $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

f) $\lim_{x\to 0}\frac{(1+x)^n-1}{x}=n$

(A) The number e called exponential number is given by e = 2.718281828 —— = 2.7183. This number e is one of the useful constants in mathematics.

(B) In calculus all logarithms are taken with respect to base 'e' that is log x=$\log_e$ x.

ILLUSTRATIVE EXAMPLES

Example 1: Evaluate: $\lim_{x\to 3}\frac{x^2-6x+9}{x-3}$, where $f(x) = \frac{x^2-6x+9}{x-3}$. Also find f (3)

Solution: At x = 3 the function is undefined as division by zero is meaningless. While taking the limit as x → 3 the function is defined near the number 3 because when x → 3 x cannot be exactly equal to 3 i.e. x – 3 ≠ 0 and consequently division by x – 3 is permissible.

Now $\lim_{x\to 3}\frac{x^2-6x+9}{x-3}=\lim_{x\to 3}\frac{(x-3)^2}{x-3}=\lim_{x\to 3}(x-3)=3-3=0.$ $f(3)=\frac{0}{0}$ is undefined

The reader may compute the left-hand and the right-hand limits as an exercise.

Example 2: A function is defined as follows:

$$f(x)=\begin{cases}-3x & \text{when } x<0\\ 2x & \text{when } x>0\end{cases}$$

Test the existence of $\lim_{x\to 0} f(x)$.

Solution: For x approaching 0 from the left x < 0.

Left-hand limit = $\lim_{x\to 0-} f(x) = \lim_{x\to 0-}(-3x) = 0$

When x approaches 0 from the right x > 0

Right-hand limit = $\lim_{x\to 0+} f(x) = \lim_{x\to 0+} 2x = 0$

Since L.H. limit = R.H. Limit, the limit exists. Thus, $\lim_{x\to 0} f(x) = 0$.

Example 3: Does $\lim_{x\to\pi} \frac{1}{\Pi - x}$ exist ?

Solution: $\lim_{x\to\pi+0} \frac{1}{\pi - x} = \to \infty$ and $\lim_{x\to\pi-0} \frac{1}{\pi - x} = +\infty$;

R.H.L. $\lim_{x\to\pi}\left(\frac{1}{\Pi - x}\right) = \lim_{h\to 0}\left[\frac{1}{\Pi-(\Pi+h}\right] = \lim_{h\to 0}\left(\frac{1}{-h}\right) \to -\infty$

Since the limits are unequal the limit does not exist.

R.H.L. = $\lim_{x\to\pi}\left(\frac{1}{\Pi - x}\right) = \lim_{h\to 0}\left[\frac{1}{\Pi-(\Pi-h}\right] = \lim_{h\to 0}\left(\frac{1}{h}\right) \to +\infty$

Example 4: : $\lim_{x\to 3} \frac{x^2+4x+3}{x^2+6x+9}$.

Solution: $\frac{x^2+4x+3}{x^2+6x+9} = \frac{x^2+3x+x+3}{(x+3)^2} = \frac{x(x+3)+1(x+3)}{(x+3)^2} = \frac{(x+3)(x+1)}{(x+3)^2} = \frac{x+1}{x+3}$

$\therefore \lim_{x\to 3} \frac{x^2+4x+3}{x^2+6x+9} = \lim_{x\to 3} \frac{x+1}{x+3} = \frac{4}{6} = \frac{2}{3}$.

Example 5: Find the following limits:

(i) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$; **(ii)** $\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$ if h > 0.

Solution:

(i) $\frac{\sqrt{x}-3}{x-9} = \frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)} = \frac{1}{\sqrt{x}+3}$. $\therefore \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$.

(ii) $\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$ $\therefore \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$

$= \frac{1}{\lim_{h\to 0}\sqrt{x+h} + \lim_{h\to 0}\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$.

Example 6: Find $\lim_{x\to 0} \frac{3x+|x|}{7x-5|x|}$.

Solution: Right-hand limit = $\lim_{x\to 0+} \frac{3x+|x|}{7x-5|x|} = \lim_{x\to 0+} \frac{3x+x}{7x-5x} = \lim_{x\to 0+} 2 = 2$

Left-hand limit $\lim_{x \to 0-} \frac{3x+|x|}{7x-5|x|} = \lim_{x \to 0-} \frac{3x-(x)}{7x-5\,(-x)} = \lim_{x \to 0-} \frac{1}{6} = \frac{1}{6}$.

Since Right-hand limit ≠ Left-hand limit the limit does not exist.

Example 7: Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$

Solution: $\lim_{x \to 0} \frac{e^x - e^{-x}}{x} = \lim_{x \to 0} \frac{(e^x-1)-(e^{-x}-1)}{x} = \lim_{x \to 0} \frac{e^x-1}{x} - \lim_{x \to 0} \frac{e^{-x}-1}{x} = 1-1=0$

Example 8: Find $\lim_{x \to \propto} \left(1+\frac{9}{x}\right)^x$. (Form 1`)

Solution: It may be noted that $\frac{x}{9}$ approaches $\propto$ as x approaches ∞. i.e. $\lim_{x \to \infty} \frac{x}{9} \to \infty$

$$\lim_{x \to \infty} \left(1+\frac{9}{x}\right)^x = \lim_{x/9 \to \propto} \left\{\left(1+\frac{1}{\frac{x}{9}}\right)^{x/9}\right\}^9$$

Substituting x/9 = z the above expression takes the form $\lim_{z \to \propto} \left\{\left(1+\frac{1}{z}\right)^z\right\}^9$

$$= \left\{\lim_{z \to \infty} \left(1+\frac{1}{z}\right)^z\right\}^9 = e^9.$$

Example 9: Evaluate: $\lim_{x \to \propto} \frac{2x+1}{x^3+1}$. $\left[\text{Form } \frac{\infty}{\infty}\right]$

Solution: As x approaches $\propto$ 2x + 1 and x^3 + 1 both approach $\propto$ and therefore the given function takes the form $\frac{\propto}{\propto}$ which is indeterminate. Therefore instead of evaluating directly let us try for suitable algebraic transformation so that the indeterminate form is avoided.

$$\lim_{x \to \propto} \frac{\frac{2}{x^2}+\frac{1}{x^3}}{1+\frac{1}{x^3}} = \frac{\lim_{x \to \propto}\left(\frac{2}{x^2}+\frac{1}{x^3}\right)}{\lim_{x \to \propto}\left(1+\frac{1}{x^3}\right)} = \frac{\lim_{x \to \propto}\frac{2}{x^2}+\lim_{x \to \propto}\frac{1}{x^3}}{\lim_{x \to \propto} 1+\lim_{x \to \propto}\frac{1}{x^3}} = \frac{0+0}{1+0} = \frac{0}{1} = 0.$$

Example 10: Find $\lim_{x\to\infty} \frac{1^2+2^2+3^2+..........+x^2}{x^3}$

Solution: $\lim_{x\to\infty} \frac{1^2+2^2+3^2+..........+x^2}{x^3}$

$$\lim_{x\to\infty} \frac{[x(x+1)(2x+1)]}{6x^3} = \frac{1}{6}\lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)\left(2+\frac{1}{x}\right)\right\}$$

$$= \frac{1}{6}\times1\times2 = \frac{1}{3}.$$

Example 11: $\lim_{x\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}......................+\frac{n}{1-n^2}\right)$

Solution : $= \lim_{x\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+\frac{3}{1-n^2}......................+\frac{n}{1-n^2}\right)$

$$= \lim_{x\to\infty} \frac{1}{1-n^2}\ (1+2+3\ ....+n)$$

$$= \lim_{x\to\infty} \frac{1}{1-n^2} \times \frac{n(n+1)}{2}$$

$$= \lim_{x\to\infty} \frac{1}{1-n^2} \times \frac{n(n+1)}{2}$$

$$= \frac{1}{2}\lim_{x\to\infty} \frac{n}{1-n}$$

$$= \frac{1}{2}\lim_{x\to\infty}\left(\frac{1}{\frac{1}{n}-1}\right)$$

$$= \frac{1}{2}\lim_{x\to\infty}\frac{1}{0-1} = \frac{1}{2}(-1) = \left(-\frac{1}{2}\right)$$

Exercise 8 (B)

Choose the most appropriate option (a) (b) (c) or (d)

1. $\lim_{x\to0} f(x)$ when f(x) = 6 is

   a) 6 b) 0 c) 1/6 d) none of these

2. $\lim_{x \to 2} (3x + 2)$ is equal to

a) 6 b) 4 c) 8 d) none of these

3. $\lim_{x \to -2} \frac{x^2-4}{x+2}$ is equal to

a) 4 b) –4 c) does not exist d) none of these

4. $\lim_{x \to \infty} \left(\frac{3}{x^2}+2\right)$

a) 0 b) 5 c) 2 d) none of these

5. $\lim_{x \to 1} \log e^{x}$ is evaluated to be

a) 0 b) e c) 1 d) none of these

6. The value of the limit of f(x) as $x \to 3$ when $f(x) = e^{x^2+2x+1}$ is

a) e^{15} b) e^{16} c) e^{10} d) none of these

7. $\lim_{x \to 1/2} \left(\frac{8x^3-1}{6x^2-5x+1}\right)$ is equal to

a) 5 b) –6 c) 6 d) none of these

8. $\lim_{x \to 0} \frac{\sqrt{1+2x^2}-\sqrt{1-2x^2}}{x^2}$ is equal to

a) 2 b) –2 c) ½ d) none of these

9. $\lim_{x \to p} \frac{\sqrt{x-q}-\sqrt{p-q}}{x^2-p^2}$ (p>q) is evaluated as

a) $\frac{1}{p\sqrt{p-q}}$ b) $\frac{1}{4p\sqrt{p-q}}$ c) $\frac{1}{2p\sqrt{p-q}}$ d) none of these

10. $\lim_{x \to 0} \frac{(3^x-1)}{x}$ is equal to

a) $10^3 \log_{10}3$ b) $\log_3 e$ c) $\log_e 3$ d) none of these

11. $\lim_{x \to 0} \frac{5^x+3^x-2}{x}$ will be equal to

a) $\log_e 15$ b) log (1/15) c) log e d) none of these

12. $\lim_{x\to 0} \dfrac{10^x - 5^x \text{-} 2^x}{x^2}$ is equal to

a) $\log_e 2 + \log_e 5$ b) $\log_e 2 \log_e 5$ c) $\log_e 10$ d) none of these

13. If $f(x) = ax^2 + bx+c$ then $\lim_{x\to 0} \dfrac{f(x+h)\text{-}f(x)}{h}$ is equal to

a) ax +b b) ax + 2b c) 2ax +b d) none of these

14. $\lim_{x\to 2} \dfrac{2x^2\text{-}7x+6}{5x^2\text{-}11x+2}$ is equal to

a) 1/9 b) 9 c) –1/9 d) none of these

15. $\lim_{x\to 1} \dfrac{x^3\text{-}5x^2+2x+2}{x^3+2x^2\text{-}6x+3}$ is equal to

a) 5 b) –5 c) 1/5 d) none of these

16. $\lim_{x\to t} \dfrac{x^3\text{-}t^3}{x^2\text{-}t^2}$ is evaluated to be

a) 3/2 b) 2/3t c) $\left(\dfrac{3}{2}\right)t$ d) none of these

17. $\lim_{x\to 0} \dfrac{4x^4+5x^3 7x^2+6x}{5x^5+7x^2+x}$ is equal to

a) 7 b) 5 c) –6 d) none of these

18. $\lim_{x\to 2} \dfrac{(x^2 - 5x + 6)(x^2 \text{-}3x +2)}{x^3\text{-}3x^2+4}$ is equal to

a) 1/3 b) 3 c) –1/3 d) none of these

19. $\lim_{x\to \propto} \dfrac{\sqrt{3x^4 + 5x^2 + 7x + 5}}{4x^2}$ is evaluated

a) $\dfrac{\sqrt{3}}{4}$ b) $\sqrt{3}$ c) –1/4 d) none of these

20. $\lim_{x\to 0} \dfrac{(e^x + e^{-x} - 2)(x^2 \text{-}3x +2)}{(x\text{-}1)}$ is equal to

a) 1 b) 0 c) –1 d) none of these

21. $\lim_{x\to 1} \frac{(1-x^{-1/3})}{(1-x^{-2/3})}$ is equal to

a) –1/2 b) 1/2 c) 2 d) none of these

22. $\lim_{x\to 4} \frac{(x^2-16)}{(x-4)}$ is evaluated as

a) 8 b) –8 c) 0 d) none of these

23. $\lim_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}$ is equal to

a) –3 b) 1/3 c) 3 d) none of these

24. $\lim_{x\to 1} \frac{x^3-1}{x-1}$ is equal to

a) 3 b) –1/3 c) –3 d) none of these

25. $\frac{(1+x)^6}{(1+x)^2-1}$ then $\lim_{x\to 0} f(x)$ is equal to

a) –1 b) 3 c) 0 d) none of these

26. $\lim_{x\to 0} \log \frac{(1+px)}{e^{3x}-1}$ is equal to

a) p/3 b) p c) 1/3 d) none of these

27. $\lim_{x\to\infty} \left(\frac{1}{x^3+x^2+x+1}\right)$ is equal to

a) 0 b) e c) $-e^6$ d) none of these

28. $\lim_{x\to\infty} \frac{2x^2+7x+5}{4x^2+3x-1}$ is equal to l where l is

a) –1/2 b) 1/2 c) 2 d) none of these

29. $\lim_{x\to\infty} \frac{(x\sqrt{x}-m\sqrt{m})}{1-x^{-2/3}}$ is equal to

a) 1 b) –1 c) 1/ 2 d) none of these

30. $\lim_{x\to 0} \frac{(x+2)^{5/3}-(p+2)^{5/3}}{x-p}$ is equal to

a) p b) 1/p c) 0 d) none of these

31. If f(x) $\frac{x^3+3x^2-9x-2}{x^3-x-6}$ and $\lim_{x\to 2}$ f(x) exists then $\lim_{x\to 2}$ (x) is equal to

a) 15/11 b) 5/11 c) 11/15 d) none of these

32. $\lim_{x\to 6} = \frac{5+2x-(3+2)}{x^2-6}$ is equal to

a) 3 – 2 b) $\frac{3-2}{2-6}$ c) $\frac{1}{2-6}$ d) none of these

33. $\lim_{x\to 2} \frac{4-x^2}{3-\sqrt{x^2+5}}$ is equal to

a) 6 b) 1/6 c) –6 d) none of these

34. $\lim_{x\to\sqrt{2}} \frac{x^{3/2}-2^{3/4}}{\sqrt{x}-2^{1/4}}$ exists and is equal to a finite value which is

a) –5 b) 1/6 c) 3√2 d) none of these

35. $\lim_{x\to 0}\left(\frac{1}{x}\right)$ log (1–x/2 is equal to

a) –1/2 b) 1/2 c) 2 d) none of these

36. $\lim_{x\to 1} \frac{(x-1)^2}{(x-1)(x^2-1)}$ is equal to

a) 1 b) 0 c) –1 d) none of these

37. $\lim_{x\to\infty}\left[\frac{1^3+2^3+3^3+--+x^3}{x^4}\right]$ is equal to

a) 1/4 b) 1/2 c) –1/4 d) none of these

8.6 CONTINUITY

By the term "continuous" we mean something which goes on without interruption and without abrupt changes. Here in mathematics the term "continuous" carries the same meaning. Thus we define continuity of a function in the following way.

A function f(x) is said to be continuous at x = a if and only if

(i) f(x) is defined at x = a

(ii) $\lim_{x\to a-}$ f(x) = $\lim_{x\to a+}$ f(x)

(iii) $\lim_{x \to a} f(x) = f(a)$

In the second condition both left-hand and right-hand limits exists and are equal.

In the third condition limiting value of the function must be equal to its functional value at x = a.

Useful Information:

(i) The sum difference and product of two continuous functions is a continuous function. This property holds good for any finite number of functions.

(ii) The quotient of two continuous functions is a continuous function provided the denominator is not equal to zero.

Example 1 : $f(x) = \frac{1}{2} - x$ when $0 < x < 1/2$

$= \frac{3}{2} - x$ when $½ < x < 1$

$= \frac{1}{2}$ when $x = \frac{1}{2}$

Discuss the continuity of f(x) at x = ½.

Solution : $\lim_{x \to \frac{1}{2}-} f(x) = \lim_{x \to \frac{1}{2}-} (1/2 - x) = 1/2 - 1/2 = 0$

$\lim_{x \to \frac{1}{2}+} f(x) = \lim_{x \to \frac{1}{2}+} (3/2 - x) = (3/2 - 1/2) = 1$

Since LHL ≠ RHL $\lim_{x \to 1/2} f(x)$ does not exist

Moreover f(1/2) = 1/2

Hence f(x) is not continuous of x = 1/2 , i.e. f (x) is discontinuous at $x = \frac{1}{2}$

Example 2 : Find the points of discontinuity of the function $f(x) = \frac{x^2+2x+5}{x^2-3x+2}$

Solution : $f(x) = \frac{x^2+2x+5}{x^2-3x+2} = \frac{x^2+2x+5}{(x-1)\,(x-2)}$

For x = 1 and x = 2 the denominator becomes zero and the function f(x) is undefined at x = 1 and x = 2. Hence the points of discontinuity are at x = 1 and x = 2.

Example 3 : A function g(x) is defined as follows:

g(x) = x when 0< x < 1

= 2 – x when x ≥ 1

Is g(x) is continuous at x = 1?

Solution :

$\lim_{x\to1-} g(x) = \lim_{x\to1-} x = 1$

$\lim_{x\to1+} g(x) = \lim_{x\to1+} (2 - x) = 2 - 1 = 1$

$\therefore \lim_{x\to1-} g(x) = \lim_{x\to1+} g(x) = 1$

Moreover g(1) = 2 –1 = 1

So $\lim_{x\to1} g(x) = g(1) = 1$

Hence f(x) is continuous at x = 1.

Example 4: The function $f(x) = (x^2 - 9) / (x - 3)$ is undefined at x = 3. What value must be assigned to f(3) if f (x) is to be continuous at x = 3?

Solution : When x approaches 3 x ≠ 3 i.e. x – 3 ≠ 0

So $\lim_{x\to3} f(x) = \lim_{x\to3} \frac{(x-3)(x+3)}{(x-3)}$

$= \lim_{x\to3} (x + 3) = 3 + 3 = 6$

Therefore if f(x) is to be continuous at x = 3, $f(3) = \lim_{x\to3} f(x) = 6$.

Example 5: Is the function f(x) = | x | continuous at x = 0?

Solution: We know | x | = x when x > 0

= 0 when x = 0

= –x when x < 0

Now $\lim_{x\to0-} f(x) = \lim_{x\to0-} (-x) = 0$ and $\lim_{x\to0+} f(x) = \lim_{x\to0+} x = 0$

Hence $\lim_{x\to0} f(x) = 0 = f(0)$

So f(x) is continuous at x = 0.

Exercise 8(C)

Choose the most appropriate option (a) (b) (c) or (d)

1. If f(x) is an odd function then

a) $\frac{f(-x)+f(x)}{2}$ is an even function

b) [| x | + 1] is even when [x] = the greater integer x ≤

c) $\frac{f(x)+f(-x)}{2}$ is neither even or odd

d) none of these.

2. If f(x) and g(x) are two functions of x such that $f(x) + g(x) = e^x$ and $f(x) - g(x) = e^{-x}$ then

a) f(x) is an odd function b) g(x) is an odd function

c) f(x) is an even function d) g(x) is an even function

3. If $f(x) = \frac{2x^2+6x-5}{12x^2+x-20}$ is to be discontinuous then

a) x = 5/4 b) x = 4/5 c) x = –4/3 d) none of these.

4. A function f(x) is defined as follows

$f(x) = x^2$ when $0 < x < 1$

$= x$ when $1 \le x < 2$

$= (1/4)\, x^3$ when $2 \le x < 3$

Now f(x) is continuous at

a) x = 1 b) x = 3 c) x = 0 d) none of these.

5. $\lim_{x\to 0} \frac{3x+|x|}{7x-5|x|}$

a) exists b) does not exist c) 1/6 d) none of these.

6. If $f(x) = \frac{(x+1)}{\sqrt{6x^2+3}+3x}$ then $\lim_{x\to -1} f(x)$ and f(-1)

a) both exists b) one exists and other does not exist

c) both do not exists d) none of these.

7. $\lim_{x\to 1} \frac{x^2-1}{\sqrt{3x+1}-\sqrt{5x-1}}$ is evaluated to be

a) 4 b) 1/4 c) –4 d) none of these.

8. $\lim (\sqrt{x+h}-\sqrt{x})$ / h where $h\to 0$ is equal to

a) 1/ 2 x b) 1/2x c) x /2 d) $\frac{1}{2\sqrt{x}}$

9. Let f(x) = x when x >0

= 0 when x = 0

= – x when x < 0

Now f(x) is

a) discontinuous at x = 0
b) continuous at x = 0
c) undefined at x = 0
d) none of these.

10. If f(x) = 5+3x for $x \geq 0$ and f(x) = 5 – 3x for $x < 0$ then f(x) is

a) continuous at x = 0

b) discontinuous and defined at x = 0

c) discontinuous and undefined at x = 0

d) none of these.

11. $\lim_{x \to 1} \left\{ \frac{(x-1)^2}{x-1} + (x^2-1) \right\}$

a) does not exist
b) exists and is equal to two
c) is equal to 1
d) none of these.

12. $\lim_{x \to 0} \frac{4^{x+1}-4}{2x}$

a) does not exist
b) exists and is equal to 4
c) exists and is equal to 4 $\log_e 2$
d) none of these.

13. Let $f(x) = \frac{(x^2-16)}{(x-4)}$ for $x \neq 4$

$= 10$ for x = 4

Then the given function is not continuous for

(a) limit f(x) does not exist

(b) limiting value of f(x) for $x \to 4$ is not equal to its function value f(4)

(c) f(x) is not defined at x = 4

(d) none of these.

14. A function f(x) is defined by f(x) = (x–2)+1 over all real values of x, now f(x) is

(a) continuous at x = 2
(b) discontinuous at x = 2
(c) undefined at x = 2
(d) none of these.

15. A function f(x) defined as follows f(x) = x+1 when $x \leq 1$

= 3 – px when $x > 1$

The value of p for which f(x) is continuous at x = 1 is

(a) –1 (b) 1 (c) 0 (d) none of these.

16. A function f(x) is defined as follows :

$f(x) = x$ when $x < 1$

$= 1+x$ when $x > 1$

$= 3/2$ when $x = 1$

Then f(x) is

(a) continuous at $x = ½$ (b) continuous at $x = 1$

(c) undefined at $x = ½$ (d) none of these.

17. Let $f(x) = x/|x|$. Now f(x) is

(a) continuous at $x = 0$ (b) discontinuous at $x = 0$

(c) defined at $x = 0$ (d) none of these.

18. $f(x) = x–1$ when $x > 0$

$= – ½$ when $x = 0$

$= x + 1$ when $x < 0$

f(x) is

(a) continuous at $x = 0$ (b) undefined at $x = 0$

(c) discontinuous at $x = 0$ (d) none of these.

19. $\lim_{x\to 0}\left(\frac{x+6}{x+1}\right)^{x+4}$ is equal to

(a) 6^4 (b) $1/e^5$ (c) $-e^5$ (d) none of these.

20. $\lim_{x\to 0}\frac{(e^{2x}-1)}{x}$ is equal to

(a) ½ (b) 2 (c) 0 (d) none of these.

21. $\lim_{x\to\infty}\frac{e^x+1}{e^x+2}$ is evaluated to be

(a) 0 (b) –1 (c) 1 (d) none of these.

22. If $\lim_{x\to 3}\left(\frac{x^n-3^n}{x-3}\right) = 108$ then the value of n is

(a) 4 (b) –4 (c) 1 (d) none of these.

23. $f(x) = (x^2 - 1) / (x^3 - 1)$ is undefined at $x = 1$ the value of f(x) at $x = 1$ such that it is continuous at $x = 1$ is

(a) 3/2 (b) 2/3 (c) – 3/2 (d) none of these.

24. $f(x) = 2x - |x|$ is

(a) undefined at $x = 0$ (b) discontinuous at $x = 0$

(c) continuous at $x = 0$ (d) none of these.

25. If f(x) = 3, when x <2

f(x) = kx^2, when x ≥2 is continuous at x = 2, then the value of k is

(a) ¾ (b) 4/3 (c) 1/3 (d) none of these.

26. $f(x) = \frac{x^2-3x+2}{x-1}$ x ≠ 1 becomes continuous at x = 1. Then the value of f(1) is

(a) 1 (b) –1 (c) 0 (d) none of these.

27. $f(x) = \frac{(x^2-2x-3)}{(x+1)}$ x ≠ –1 and f(x) = k, when x = –1 If(x) is continuous at x= –1 .

The value of k will be

(a) –1 (b) 1 (c) –4 (d) none of these.

28. $\lim_{x\to1}\left(\frac{x^2-\sqrt{x}}{\sqrt{x}-1}\right)$ is equal to

(a) 3 (b) –3 (c) 1/3 (d) none of these.

29. $\lim_{x\to0}\frac{e^{x^2}-1}{x^2}$ is evaluated to be

(a) 1 (b) ½ (c) –1 (d) none of these.

30. If $\lim_{x\to2}\frac{x^n-2^n}{x-2}$ = 80 and n is a positve integer, then

(a) n = 5 (b) n = 4 (c) n = 0 (d) none of these.

31. $\lim_{x\to\sqrt{2}}\frac{x^{5/2}-2^{5/4}}{\sqrt{x}-2^{1/4}}$ is equal to

(a) 1/ 10 (b) 10 (c) 20 (d) none of these.

32. $\lim_{x\longrightarrow1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$ is evaluated to be

(a) 1/9 (b) 9 (c) – 1/9 (d) none of these.

33. $\lim_{n\to\infty}\left[\frac{1}{6}+\frac{1}{6^2}+\frac{1}{6^3}+\cdots\cdots\cdots\cdots+\frac{1}{6^n}\right]$ is

(a) 1/5 (b) 1/6 (c) – 1/5 (d) none of these.

34. The value of $\lim_{x \to 0} u^x + v^x + w^x - 3 / x$ is

(a) uvw (b) log uvw (c) log (1/uvw) (d) none of these.

35. $\lim_{x \to 0} \dfrac{x}{\log(1+x)}$ is equal to

(a) 1 (b) 2 (c) –0.5 (d) none of these.

ANSWERS

Exercise 8(A)

1. a	2. b	3. c	4. a	5. c	6. a	7. b	8. c
9. b	10. a						

Exercise 8(B)

1. a	2. c	3. b	4. c	5. c	6. b	7. c	8. a
9. b	10. c	11. a	12. d	13. c	14. a	15. b	16. c
17. a	18. c	19. a	20. b	21. b	22. a	23. c	24. a
25. b	26. a	27. a	28. b	29. a	30. d	31. a	32. c
33. a	34. c	35. a	36. b	37. a			

Exercise 8(C)

1. a	2. bc	3. a,c	4. a	5. a	6. b	7. c	8. d
9. b	10. a	11. b	12. c	13. b	14. a	15. b	16. a
17. b	18. c	19. a	20. b	21. c	22. a	23. b	24. c
25. a	26. b	27. c	28. a	29. a	30. a	31. b	32. c
33. a	34. b	35. a					

ADDITIONAL QUESTION BANK

1. The value of the limit when *n* tends to infinity of the expression $(7n^3-8n^2+10n-7)\div(8n^3-9n^2+5)$ is

   (A) 7/8 (B) 8/7 (C) 1 (D) None

2. The value of the limit when *n* tends to infinity of the expression $(n^4-7n^2+9)\div(3n^2+5)$ is

   (A) 0 (B) 1 (C) –1 (D) ∝

3. The value of the limit when *n* trends to infinity of the expression $(3n^3+7n^2-11n+19)\div(17n^4+18n^3-20n+45)$ is

   (A) 0 (B) 1 (C) –1 (D) $1/\sqrt{2}$

4. The value of the limit when *n* tends to infinity of the expression $(2n)\div[(2n-1)(3n+5)]$ is

   (A) 0 (B) 1 (C) –1 (D) $1/\sqrt{2}$

5. The value of the limit when *n* tends to infinity of the expression $n^{1/3}(n^2+1)^{1/3}(2n^2+3n+1)^{-1/2}$ is

   (A) 0 (B) 1 (C) –1 (D) $1/\sqrt{2}$

6. The value of the limit when *x* tends to *a* of the expression $(x^n-a^n)\div(x-a)$ is

   (A) na^{n-1} (B) na^n (C) $(n-1)a^{n-1}$ (D) $(n+1)a^{n+1}$

7. The value of the limit when *x* tends to zero of the expression $(1+n)^{1/n}$ is

   (A) *e* (B) 0 (C) 1 (D) –1

8. The value of the limit when *n* tends to infinity of the expression $\left(1+1/n\right)^n$ is

   (A) *e* (B) 0 (C) 1 (D) –1

9. The value of the limit when *x* tends to zero of the expression $[(1+x)^n-1]\div x$ is

   (A) *n* (B) *n* + 1 (C) *n* – 1 (D) *n*(*n* – 1)

10. The value of the limit when *x* tends to zero of the expression $(e^x-1)/x$ is

    (A) 1 (B) 0 (C) – 1 (D) indeterminate

11. The value of the limit when *x* tends to 3 of the expression $(x^2+2x-15)/(x^2-9)$ is

    (A) 4/3 (B) 3/4 (C) 1/2 (D) indeterminate

12. The value of the limit when x tends to zero of the expression $[(a+x^2)^{1/2}-(a-x^2)^{1/2}]\div x^2$ is

(A) $a^{-1/2}$ (B) $a^{1/2}$ (C) a (D) a^{-1}

13. The value of the limit when x tends to unity of the expression $[(3+x)^{1/2}-(5-x)^{1/2}]\div(x^2-1)$ is

(A) 1/4 (B) ½ (C) –1/4 (D) –1/2

14. The value of the limit when x tends to 2 of the expression $(x-2)^{-1}-(x^2-3x+2)^{-1}$ is

(A) 1 (B) 0 (C) –1 (D) None

15. The value of the limit when n tends to infinity of the expression $2^{-n}(n^2+5n+6)[(n+4)(n+5)]^{-1}$ is

(A) 1 (B) 0 (C) –1 (D) None

16. The value of $\lim_{n\to\infty} \frac{n+1}{n^2}\div\frac{1}{n}$

(A) 1 (B) 0 (C) –1 (D) None

17. Find $\lim_{n\to\infty}[n^{1/2}+(n+1)^{1/2}]^{-1}\div n^{-1/2}$

(A) 1/2 (B) 0 (C) 1 (D) None

18. Find $\lim_{n\to\infty}(2n-1)(2n)n^2(2n+1)^{-2}(2n+2)^{-2}$

(A) 1/4 (B) 1/2 (C) 1 (D) None

19. Find $\lim_{n\to\infty}[(n^3+1)^{1/2}-n^{3/2}]\div n^{3/2}$

(A) 1/4 (B) 0 (C) 1 (D) None

20. Find $\lim_{n\to\infty}[(n^4+1)^{1/2}-(n^4-1)^{1/2}]\div n^{-2}$

(A) 1/4 (B) 1/2 (C) 1 (D) None

21. Find $\lim_{n\to\infty}(2^n-2)(2^n+1)^{-1}$

(A) 1/4 (B) 1/2 (C) 1 (D) None

22. Find $\lim_{n \to \infty} n^n (n+1)^{-n-1} \div n^{-1}$

(A) e^{-1} (B) e (C) 1 (D) –1

23. Find $\lim_{n \to \infty} (2n-1)2^n (2n+1)^{-1} 2^{1-n}$

(A) 2 (B) 1/2 (C) 1 (D) None

24. Find $\lim_{n \to \infty} 2^{n-1}(10+n)(9+n)^{-1} 2^{-n}$

(A) 2 (B) 1/2 (C) 1 (D) None

25. Find $\lim_{n \to \infty} [n(n+2)] \div (n+1)^2$

(A) 2 (B) 1/2 (C) 1 (D) None

26. Find $\lim_{n \to \infty} [n! 3^{n+1}] \div [3^n (n+1)!]$

(A) 0 (B) 1 (C) –1 (D) 2

27. Find $\lim_{n \to \infty} (n^3+a)[(n+1)^3 a]^{-1}(2^{n+1}+a)(2^n+a)^{-1}$

(A) 0 (B) 1 (C) –1 (D) 2

28. Find $\lim_{n \to \infty} (n^2+1)[(n+1)^2+1]^{-1} 5^{n+1} 5^{-n}$

(A) 5 (B) e^{-1} (C) 0 (D) None

29. Find $\lim_{n \to \infty} [n^n.(n+1)!] \div [n!(n+1)^{n+1}]$

(A) 5 (B) e^{-1} (C) 0 (D) None

30. Find $\lim_{n \to \infty} [\{1.3.5....(2n-1)\}(n+1)^4] \div [n^4 \{1.3.5....(2n-1)(2n+1)\}]$

(A) 5 (B) e^{-1} (C) 0 (D) None

31. Find $\lim_{n \to \infty} [x^n.(n+1)] \div [n x^{n+1}]$

(A) x^{-1} (B) x (C) 1 (D) None

32. Find $\lim_{n \to \infty} n^n (1+n)^{-n}$

(A) e^{-1} (B) e (C) 1 (D) –1

33. Find $\lim_{n \to \infty} [(n+1)^{n+1}.n^{-n-1}-(n+1).n^{-1}]^{-n}$

(A) $(e-1)^{-1}$ (B) $(e+1)^{-1}$ (C) e-1 (D) e+1

34. Find $\lim_{n \to \infty} (1+n^{-1})[1+(2n)^{-1}]^{-1}$

(A) 1/2 (B) 3/2 (C) 1 (D) –1

35. Find $\lim_{n \to \infty} [4n^2+6n+2]\div 4n^2$

(A) 1/2 (B) 3/2 (C) 1 (D) –1

36. $3x^2+2x-1$ is continuous

(A) at x = 2 (B) for every value of x
(C) both (A) and (B) (D) None

37. $f(x) = \frac{|x|}{x}$, when x ≠ 0, then f(x) is

(A) discontinuous at x = 0 (B) continuous at x = 0
(C) maxima at x = 0 (D) minima at x = 0

38. $e^{-1/x}[1+e^{1/x}]^{-1}$ is

(A) discontinuous at x = 0 (B) continuous at x = 0
(C) maxima at x = 0 (D) minima at x = 0

39. If $f(x)=(x^2-4)\div(x-2)$ for $x<2$, $f(x)=4$ for $x=2$ and $f(x)=2$ for $x>2$, then f(x) at x = 2 is

(A) discontinuous (B) continuous
(C) maxima (D) minima

40. If $f(x)=x$ for $0 \leq x<1/2$, $f(x)=1$ for $x=1/2$ and $f(x)=1-x$ for $1/2<x<1$ then at the function is

(A) discontinuous (B) continuous
(C) left-hand limit coincides with f(1/2) (D) right-hand limit coincides with left-hand limit.

41. If $f(x)=9x \div (x+2)$ for $x<1$, $f(1)=3$, $f(x)=(x+3)x^{-1}$ for $x>1$, then in the interval (–3, 3) the function is

(A) continuous at x = –2

(B) continuous at x = 1

(C) discontinuous for values of x other than –2 1 in the interval

(D) None

ANSWERS

1)	A	2)	D	3)	A	4)	A	5)	D	6)	A
7)	A	8)	A	9)	A	10)	A	11)	A	12)	A
13)	A	14)	A	15)	B	16)	A	17)	A	18)	A
19)	B	20)	C	21)	C	22)	A	23)	A	24)	B
25)	C	26)	A	27)	D	28)	A	29)	B	30)	C
31)	A	32)	A	33)	A	34)	A	35)	C	36)	C
37)	A	38)	A	39)	A	40)	A	41)	D		